

Ejercicios de cálculo de derivadas

1 Calcula las derivadas de las funciones:

1 $f(x) = 5$

2 $f(x) = -2x$

3 $f(x) = -2x + 2$

4 $f(x) = -2x^2 - 5$

5 $f(x) = 2x^4 + x^2 - x^2 + 4$

6 $f(x) = \frac{x^3 + 2}{3}$

7 $f(x) = \frac{1}{3x^2}$

8 $f(x) = \frac{x+1}{x-1}$

9 $f(x) = (5x^2 - 3) \cdot (x^2 + x + 4)$

2 Calcula mediante la fórmula de la derivada de una potencia:

1 $f(x) = \frac{5}{x^5}$

2 $f(x) = \frac{5}{x^5} + \frac{3}{x^2}$

3 $f(x) = \sqrt{x}$

4 $f(x) = \frac{1}{\sqrt{x}}$

5 $f(x) = \frac{1}{x\sqrt{x}}$

6 $f(x) = \sqrt[3]{x^2} + \sqrt{x}$

7 $f(x) = (x^2 + 3x - 2)^4$

3 Calcula mediante la fórmula de la derivada de una raíz:

1 $f(x) = \sqrt{x^2 - 2x + 3}$

2 $f(x) = \sqrt[4]{x^5 - x^3 - 2}$

3 $f(x) = \sqrt[3]{\frac{x^2 + 1}{x^2 - 1}}$

4 Deriva las funciones exponenciales

1 $f(x) = 10^{\sqrt{x}}$

2 $f(x) = e^{3-x^2}$

3 $f(x) = \frac{e^x + e^{-x}}{2}$

4 $f(x) = 3^{2x^2} \cdot \sqrt{x}$

5 $f(x) = \frac{e^{2x}}{x^2}$

5 Calcula la derivada de la funciones logarítmicas:

1 $f(x) = \ln(2x^4 - x^3 + 3x^2 - 3x)$

2 $f(x) = \ln\left(\frac{e^x + 1}{e^x - 1}\right)$

3 $f(x) = \log \sqrt{\frac{1+x}{1-x}}$

4 $f(x) = \ln \sqrt{x(1-x)}$

5 $f(x) = \ln \sqrt[3]{\frac{3x}{x+2}}$

6 Calcula la derivada de la funciones trigonométricas:

1 $f(x) = \operatorname{sen} \frac{1}{2} x$

2 $f(x) = \cos(7 - 2x)$

3 $f(x) = 3 \operatorname{tg} 2x$

4 $f(x) = \sec(5x + 2)$

5 $f(x) = \sqrt[3]{\operatorname{sen} x}$

6 $f(x) = \operatorname{sen}^3 3x$

7 $f(x) = \operatorname{cotg}(3 - 2x)$

8 $f(x) = \cos \frac{x+1}{x-1}$

9 $f(x) = \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}$

7 Calcula la derivada de la funciones trigonométricas inversas:

1 $f(x) = \operatorname{arc} \operatorname{sen}(1 - 2x^2)$

2 $f(x) = \operatorname{arc} \operatorname{sen} \sqrt{x^2 - 4}$

3 $f(x) = \operatorname{arc} \operatorname{cose} e^x$

4 $f(x) = \operatorname{arc} \operatorname{tg} \sqrt{x}$

5 $f(x) = \operatorname{arctg} \frac{1+x}{1-x}$

11 Derivar implícitamente:

1 $x^2 y - x y^2 + y^2 = 7$

2 $x^2 \operatorname{sen}(x + y) - 5y e^x = 3$

Soluciones:

1

Calcula las derivadas de las funciones:

1 $f(x) = 5$

$$f'(x) = 0$$

2 $f(x) = -2x$

$$f'(x) = -2$$

3 $f(x) = -2x + 2$

$$f'(x) = -2$$

4 $f(x) = -2x^2 - 5$

$$f'(x) = -4x$$

5 $f(x) = 2x^4 + x^3 - x^2 + 4$

$$f'(x) = 8x^3 + 3x^2 - 2x$$

6 $f(x) = \frac{x^3 + 2}{3}$

$$f'(x) = x^2$$

$$7 \quad f(x) = \frac{1}{3x^2}$$

$$f'(x) = \frac{-6x}{(3x)^2} = \frac{-6x}{9x^4} = -\frac{2}{3x^3}$$

$$8 \quad f(x) = \frac{x+1}{x-1}$$

$$f'(x) = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$9 \quad f(x) = (5x^2 - 3) \cdot (x^2 + x + 4)$$

$$f'(x) = 10x(x^2 + x + 4) + (5x^2 - 3)(2x + 1) = 20x^3 + 15x^2 + 34x - 3$$

2

Calcula mediante la fórmula de la derivada de una potencia:

$$1 \quad f(x) = \frac{5}{x^5} = 5x^{-5}$$

$$f'(x) = -25x^{-6} = -\frac{25}{x^6}$$

$$2 \quad f(x) = \frac{5}{x^5} + \frac{3}{x^2} = 5x^{-5} + 3x^{-2}$$

$$f'(x) = -25x^{-6} - 6x^{-3} = -\frac{25}{x^6} - \frac{6}{x^3}$$

$$3 \quad f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$4 \quad f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2} x^{-\frac{1}{2}-1} = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}}$$

$$5 \quad f(x) = \frac{1}{x\sqrt{x}} = \frac{1}{x \cdot x^{\frac{1}{2}}} = x^{-\frac{3}{2}}$$

$$f'(x) = -\frac{3}{2} x^{-\frac{5}{2}} = -\frac{3}{2\sqrt{x^5}}$$

$$6 \quad f(x) = \sqrt[3]{x^2} + \sqrt{x} = x^{\frac{2}{3}} + x^{\frac{1}{2}}$$

$$f'(x) = \frac{2}{3} x^{\frac{2}{3}-1} + \frac{1}{2} x^{\frac{1}{2}-1} = \frac{2}{3} x^{-\frac{1}{3}} + \frac{1}{2} x^{-\frac{1}{2}} = \frac{2}{3\sqrt[3]{x}} + \frac{1}{2\sqrt{x}}$$

$$7 \quad f(x) = (x^2 + 3x - 2)^4$$

$$f'(x) = x^4 (x^2 + 3x - 2)^3 (2x + 3)$$

3

Calcula mediante la fórmula de la derivada de una raíz:

$$1 \quad f(x) = \sqrt{x^2 - 2x + 3}$$

$$f'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x + 3}} = \frac{x - 1}{\sqrt{x^2 - 2x + 3}}$$

$$2 \quad f(x) = \sqrt[4]{x^5 - x^3 - 2}$$

$$f'(x) = \frac{5x^4 - 3x^2}{4\sqrt[4]{(x^5 - x^3 - 2)^3}}$$

$$3 \quad f(x) = \sqrt[3]{\frac{x^2+1}{x^2-1}}$$

$$f'(x) = \frac{2x(x^2-1) - (x^2+1)2x}{(x^2-1)^2} = \frac{-4x}{3 \sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}$$

$$= \frac{\frac{-4x}{(x^2-1)^2}}{3 \sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}} = \frac{-4x}{3(x^2-1)^2 \sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}$$

$$\frac{-4x}{3 \sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2} (x^2-1)^4} = \frac{-4x}{3 \sqrt[3]{(x^2+1)^2 (x^2-1)^2}}$$

$$= \frac{-4x}{3 \sqrt[3]{(x^4-1)^2}}$$

4

Deriva las funciones exponenciales:

$$1 \quad f(x) = 10^{\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot 10^{\sqrt{x}} \cdot \ln 10$$

$$2 \quad f(x) = e^{3-x^2}$$

$$f'(x) = -2x \cdot e^{3-x^2}$$

$$3 \quad f(x) = \frac{e^x + e^{-x}}{2}$$

$$f'(x) = \frac{e^x - e^{-x}}{2}$$

$$4 \quad f(x) = 3^{2x^2} \cdot \sqrt{x}$$

$$f'(x) = 4x \cdot 3^{2x^2} \cdot \ln 3 \cdot \sqrt{x} + \frac{3^{2x^2}}{2\sqrt{x}} =$$

$$= 3^{2x^2} \left(4x \cdot \sqrt{x} \cdot \ln 3 + \frac{1}{2\sqrt{x}} \right)$$

$$5 \quad f(x) = \frac{e^{2x}}{x^2}$$

$$f'(x) = \frac{2 \cdot e^{2x} \cdot x^2 - e^{2x} \cdot 2x}{x^4} = \frac{2x \cdot e^{2x} (x - 1)}{x^4} =$$

$$= \frac{2 \cdot e^{2x} (x - 1)}{x^3}$$

5

Calcula la derivada de la funciones logarítmicas:

$$1 \quad f(x) = \ln(2x^4 - x^3 + 3x^2 - 3x)$$

$$f'(x) = \frac{8x^3 - 3x^2 + 6x - 3}{2x^4 - x^3 + 3x^2 - 3x}$$

$$2 \quad f(x) = \ln\left(\frac{e^x + 1}{e^x - 1}\right)$$

Aplicando las **propiedades de los logaritmos** obtenemos:

$$f(x) = \ln(e^x + 1) - \ln(e^x - 1)$$

$$f'(x) = \frac{e^x}{e^x + 1} - \frac{e^x}{e^x - 1} = \frac{e^{2x} - e^x - e^{2x} - e^x}{(e^x + 1)(e^x - 1)} =$$

$$= \frac{-2e^x}{e^{2x} - 1}$$

3 $f(x) = \log \sqrt{\frac{1+x}{1-x}}$

Aplicando las **propiedades de los logaritmos** obtenemos:

$$f(x) = \frac{1}{2} [\log(1+x) - \log(1-x)]$$

$$f'(x) = \frac{1}{2} \left(\frac{1}{1+x} - \frac{-1}{1-x} \right) \cdot \log e = \frac{1}{2} \cdot \frac{1-x+1+x}{1-x^2} \cdot \log e =$$

$$= \frac{2}{1-x^2} \cdot \log e$$

4 $f(x) = \ln \sqrt{x(1-x)}$

Aplicando las **propiedades de los logaritmos** obtenemos:

$$f(x) = \frac{1}{2} [\ln x + \ln(1-x)]$$

$$f'(x) = \frac{1}{2} \left(\frac{1}{x} + \frac{-1}{1-x} \right) = \frac{1}{2} \cdot \frac{1-x-x}{x(1-x)} =$$

$$= \frac{1-2x}{2x(1-x)}$$

5 $f(x) = \ln \sqrt[3]{\frac{3x}{x+2}}$

Aplicando las **propiedades de los logaritmos** obtenemos:

$$f(x) = \frac{1}{3} [\ln 3x - \ln(x+2)]$$

$$f'(x) = \frac{1}{3} \left(\frac{3}{3x} - \frac{1}{x+2} \right) = \frac{1}{3} \cdot \frac{x+2-x}{x(x+2)} =$$

$$= \frac{2}{3x(x+2)}$$

6

Calcula la derivada de la funciones trigonométricas:

1 $f(x) = \operatorname{sen} \frac{1}{2} x$

$$f'(x) = \frac{1}{2} \cos \frac{1}{2} x$$

2 $f(x) = \cos(7 - 2x)$

$$f'(x) = -(-2) \cdot \operatorname{sen}(7 - 2x) = 2 \cdot \operatorname{sen}(7 - 2x)$$

3 $f(x) = 3 \operatorname{tg} 2x$

$$f'(x) = 6(1 + \operatorname{tg}^2 2x)$$

4 $f(x) = \sec(5x + 2)$

$$f'(x) = 5 \operatorname{tg}(5x + 2) \cdot \sec(5x + 2)$$

5 $f(x) = \sqrt[3]{\operatorname{sen} x}$

$$f'(x) = \frac{\cos x}{3\sqrt[3]{\operatorname{sen}^2 x}}$$

6 $f(x) = \operatorname{sen}^3 3x$

$$f'(x) = 3 \cdot \operatorname{sen}^2 3x \cdot 3 \cdot \cos 3x = 9 \cdot \operatorname{sen}^2 3x \cdot \cos 3x$$

7 $f(x) = \operatorname{cotg}(3 - 2x)$

$$f'(x) = \frac{2}{\operatorname{sen}^2(3 - 2x)}$$

$$8 \quad f(x) = \cos \frac{x+1}{x-1}$$

$$f'(x) = -\frac{x-1-(x+1)}{(x-1)^2} \operatorname{sen} \frac{x+1}{x-1} = \frac{2}{(x-1)^2} \cdot \operatorname{sen} \frac{x+1}{x-1}$$

$$9 \quad f(x) = \sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}$$

$$f'(x) = \frac{1}{2\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}} \cdot \frac{-\cos x(1+\operatorname{sen} x) - (1+\operatorname{sen} x)\cos x}{(1+\operatorname{sen} x)^2} =$$

$$= \frac{1}{2\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}} \cdot \frac{-\cos x - \operatorname{sen} x \cdot \cos x - \cos x + \operatorname{sen} x \cdot \cos x}{(1+\operatorname{sen} x)^2} =$$

$$= \frac{1}{2\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}} \cdot \frac{-2 \cos x}{(1+\operatorname{sen} x)^2} = \frac{-2 \cos x}{2\sqrt{\frac{(1-\operatorname{sen} x)(1+\operatorname{sen} x)^4}{1+\operatorname{sen} x}}} =$$

$$= -\frac{\cos x}{\sqrt{(1-\operatorname{sen} x)(1+\operatorname{sen} x)^3}} = -\frac{\cos x}{\sqrt{(1-\operatorname{sen} x)(1+\operatorname{sen} x)(1+\operatorname{sen} x)^2}} =$$

$$= -\frac{\cos x}{\sqrt{1-\operatorname{sen} x} \cdot (1+\operatorname{sen} x)} = -\frac{\cos x}{\cos x \cdot (1+\operatorname{sen} x)} =$$

$$= -\frac{1}{1+\operatorname{sen} x}$$

7

Calcula la derivada de la funciones trigonométricas inversas:

$$1 \quad f(x) = \operatorname{arc} \operatorname{sen}(1-2x^2)$$

$$f'(x) = \frac{-4x}{\sqrt{1-(1-2x^2)^2}}$$

$$2 \quad f(x) = \operatorname{arc} \operatorname{sen} \sqrt{x^2-4}$$